

Apoastron Shift Constraints on Dark Matter Distribution at the Galactic Center

A.F. Zakharov*

*National Astronomical Observatories of Chinese
Academy of Sciences, Beijing 100012, China[†]*

A.A. Nucita[‡]

*Department of Physics and INFN, University of Lecce, CP 193, I-73100 Lecce, Italy;
XMM-Newton Science Operations Centre, ESAC,
ESA, PO Box 50727, 28080 Madrid, Spain*

F. De Paolis[§] and G. Ingrosso[¶]

Department of Physics and INFN, University of Lecce, CP 193, I-73100 Lecce, Italy

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Abstract

The existence of dark matter (DM) at scales of few pc down to $\simeq 10^{-5}$ pc around the centers of galaxies and in particular in the Galactic Center region has been considered in the literature. Under the assumption that such a DM clump, principally constituted by non-baryonic matter (like WIMPs) does exist at the center of our galaxy, the study of the γ -ray emission from the Galactic Center region allows us to constrain both the mass and the size of this DM sphere. Further constraints on the DM distribution parameters may be derived by observations of bright infrared stars around the Galactic Center. Hall and Gondolo [1] used estimates of the enclosed mass obtained in various ways and tabulated by Ghez et al. [2, 3]. Moreover, if a DM cusp does exist around the Galactic Center it could modify the trajectories of stars moving around it in a sensible way depending on the DM mass distribution. Here, we discuss the constraints that can be obtained with the orbit analysis of stars (as S2 and S16) moving inside the DM concentration with present and next generations of large telescopes. In particular, consideration of the S2 star apoastron shift may allow improving limits on the DM mass and size.

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*Electronic address: zakharov@itep.ru

†Also at Institute of Theoretical and Experimental Physics, Moscow, 117259, Russia;

CAMP, National University of Sciences and Technology, Rawalpindi, Pakistan;

BLTP, Joint Institute for Nuclear Research, Dubna, Russia

‡Electronic address: anucita@sciops.esa.int

§Electronic address: Francesco.DePaolis@le.infn.it

¶Electronic address: Gabriele.Ingrosso@le.infn.it

I. INTRODUCTION

In the last years intensive searches for dark matter (DM), especially its non-baryonic component, both in galactic halos and at galaxy centers have been undertaken (see for example [4, 5] for recent results). It is generally accepted that the most promising candidate for the DM non-baryonic component is neutralino. In this case, the γ -flux from galactic halos (and from our Galactic halo in particular) could be explained by neutralino annihilation [6, 7, 8, 9, 10, 11, 12, 13]. Since γ -rays are detected not only from high galactic latitude, but also from the Galactic Center, there is a wide spread hypothesis (see [14] for a discussion) that a DM concentration might be present at the Galactic Center. In this case the Galactic Center could be a strong source of γ -rays and neutrinos [4, 7, 15, 16, 17, 18, 19, 20, 21, 22] due to DM annihilation. Since it is also expected that DM forms spikes at galaxy centers [23, 24, 25] the γ -ray flux from the Galactic Center should increase significantly in that case.

At the same time, progress in monitoring bright stars near the Galactic Center have been reached recently [2, 3, 26]. The astrometric limit for bright stellar sources near the Galactic Center with 10 meter telescopes is today $\delta\theta_{10} \sim 1$ mas and the Next Generation Large Telescope (NGLT) will be able to improve this number at least down to $\delta\theta_{30} \sim 0.5$ mas [28, 29] or even to $\delta\theta_{30} \sim 0.1$ mas [27, 28, 29] in the K-band. Therefore, it will be possible to measure the proper motion for about ~ 100 stars with astrometric errors several times smaller than errors in current observations.

The aim of this paper is to constrain the parameters of the DM distribution possible present around the Galactic Center by considering the induced apoastron shift due to the presence of this DM sphere and either available data obtained with the present generation of telescopes (the so called *conservative* limit) and also expectations from future NGLT observations or with other advanced observational facilities.

II. THE MASS CONCENTRATION AT THE GALACTIC CENTER

Recent advancements in infrared astronomy are allowing to test the scale of the mass profile at the center of our galaxy down to tens of AU. With the Keck 10 m telescope, the proper motion of several stars orbiting the Galactic Center black hole have been monitored and almost entire orbits, as for example that of the S2 star, have been measured allowing an

unprecedented description of the Galactic Center region. Measurements of the amount of mass $M(< r)$ contained within a distance r from the Galactic Center are continuously improved as more precise data are collected. Recent observations [2] extend down to the periastron distance ($\simeq 3 \times 10^{-4}$ pc) of the S16 star and they correspond to a value of the enclosed mass within $\simeq 3 \times 10^{-4}$ pc of $\simeq 3.67 \times 10^6 M_\odot$. Several authors have used these observations to model the Galactic Center mass concentration. Here and in the following, we use the three component model for the central region of our galaxy based on estimates of enclosed mass given by Ghez et al [2, 3] recently proposed [1]. This model is constituted by the central black hole, the central stellar cluster and the DM sphere (made of WIMPs), i.e.

$$M(< r) = M_{BH} + M_*(< r) + M_{DM}(< r) , \quad (1)$$

where M_{BH} is the mass of the central black hole Sagittarius A*. For the central stellar cluster, the empirical mass profile is

$$M_*(< r) = \begin{cases} M_* \left(\frac{r}{R_*} \right)^{1.6} , & r \leq R_* \\ M_* \left(\frac{r}{R_*} \right)^{1.0} , & r > R_* \end{cases} \quad (2)$$

with a total stellar mass $M_* = 0.88 \times 10^6 M_\odot$ and a size $R_* = 0.3878$ pc.

As far as the mass profile of the DM concentration is concerned, Hall and Gondolo [1] have assumed a mass distribution of the form

$$M_{DM}(< r) = \begin{cases} M_{DM} \left(\frac{r}{R_{DM}} \right)^{3-\alpha} , & r \leq R_{DM} \\ M_{DM} , & r > R_{DM} \end{cases} \quad (3)$$

M_{DM} and R_{DM} being the total amount of DM in the form of WIMPs and the radius of the spherical mass distribution, respectively.

Hall and Gondolo [1] discussed limits on DM mass around the black hole at the Galactic Center. It is clear that present observations of stars around the Galactic Center do not exclude the existence of a DM sphere with mass $\simeq 4 \times 10^6 M_\odot$, well contained within the orbits of the known stars, if its radius R_{DM} is $\lesssim 2 \times 10^{-4}$ pc (the periastron distance of the S16 star in the more recent analysis [3]). However, if one considers a DM sphere with larger radius, the corresponding upper value for M_{DM} decreases (although it tends again to

increase for extremely extended DM configurations with $R_{DM} \gg 10$ pc). In the following, we will assume for definiteness a DM mass $M_{DM} \sim 2 \times 10^5 M_\odot$, that is the upper value for the DM sphere in [1] within an acceptable confidence level in the range $10^{-3} - 10^{-2}$ pc for R_{DM} . As it will be clear in the following, we emphasize that even a such small value for the DM mass (that is about only 5% of the standard estimate $3.67 \pm 0.19 \times 10^6 M_\odot$ for the dark mass at the Galactic Center [3]) may give some observational signatures.

Evaluating the S2 apoastron shift [38] as a function of R_{DM} , one can further constrain the DM sphere radius since even now we can say that there is no evidence for negative apoastron shift for the S2 star orbit at the level of about 10 mas [26]. In addition, since at present the precision of the S2 orbit reconstruction is about 1 mas, we can say that even without future upgrades of the observational facilities and simply monitoring the S2 orbit, it will be possible within about 15 years to get much more severe constraints on R_{DM} .

Moreover, observational facilities will allow in the next future to monitor faint infrared objects at the astrometric precision of about $10 \mu\text{as}$ [30] and, in this case, previous estimates will be sensibly improved since it is naturally expected to monitor eccentric orbits for faint infrared stars closer to the Galactic Center with respect to the S2 star.

In Fig. 1, the mass profile $M(< r)$ [2] obtained by using observations of stars nearby the Galactic Center is shown (solid line). The dotted line represents the stellar mass profile as given in Eq. (2), while the dashed lines are for DM spheres with mass $M_{DM} \simeq 2 \times 10^5 M_\odot$ and radii $R_{DM} = 10^{-3}$ and 10^{-2} pc, respectively.

In the following section, we study the motion of stars as a consequence of the gravitational potential $\Phi(r)$ due the mass profile given in Eq. (1). As usual, the gravitational potential can be evaluated as

$$\Phi(r) = -G \int_r^\infty \frac{M(r')}{r'^2} dr' . \quad (4)$$

For convenience, in Fig. 2 the gravitational potential due to the total mass (solid line) contained within r is given as function of the galactocentric distance. For comparison, the contributions due to the single mass components, i.e. central black hole, stellar cluster and DM sphere, are also shown.

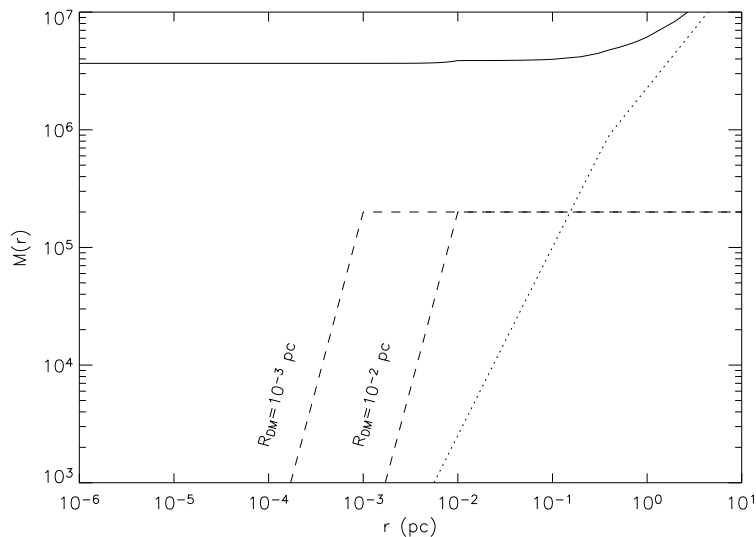


FIG. 1: The mass $M(< r)$ obtained in [2] from observations of stars at the Galactic Center is shown (solid line). The dotted line represents the stellar mass profile as given in Eq. (2), while the dashed lines are for DM spheres with radii $R_{DM} = 10^{-3}$ and 10^{-2} pc and mass $M_{DM} \simeq 2 \times 10^5 M_{\odot}$, that corresponds to some acceptable estimate for the upper limit of M_{DM} from Figs. 4-6 in [1] for R_{DM} in the above range of values.

III. APOASTRON SHIFT CONSTRAINTS

According to GR, the motion of a test particle can be fully described by solving the geodesic equations. Under the assumption that the matter distribution is static and pressureless, the equations of motion at the first post-Newtonian (PN) approximation become (see e.g. [31, 32, 36])

$$\frac{d\mathbf{v}}{dt} \simeq -\nabla(\Phi_N + 2\Phi_N^2) + 4\mathbf{v}(\mathbf{v} \cdot \nabla)\Phi_N - v^2\nabla\Phi_N. \quad (5)$$

We note that the PN-approximation is the first relativistic correction from which the apoastron advance phenomenon arises [39]. In the case of the S2 star, the apoastron shift as seen from Earth (from Eq. (7)) due to the presence of a central black hole is about 1 mas, therefore not directly detectable at present since the available precision in the apoastron shift is about 10 mas (but it will become about 1 mas in 10–15 years even without considering possible technological improvements). It is also evident that higher order relativistic corrections to the S2 apoastron shift are even smaller and therefore may be neglected at

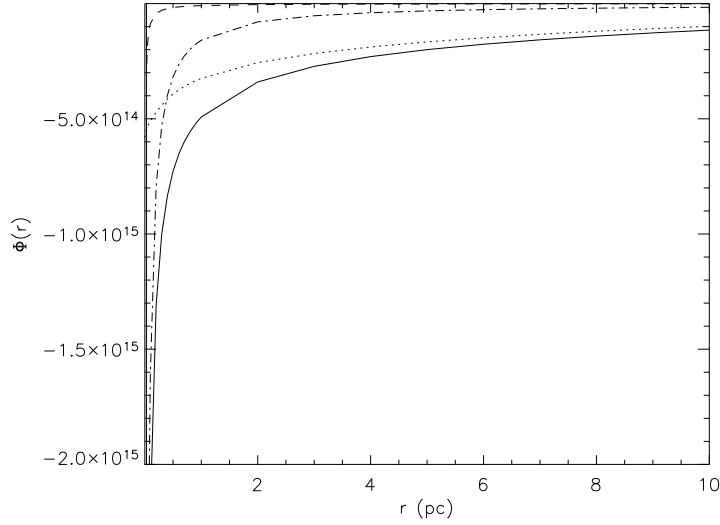


FIG. 2: The gravitational potential $\Phi(r)$ (solid line) in cgs units as a function of the galactocentric distance r as due to the mass $M(r)$ in Eq.(1) is shown. For comparison, also the gravitational potentials due to the single mass components, i.e. black hole (dashed line), stellar cluster (dot-dashed line) and DM (dotted line), are also given. Here we assume that DM mass $M_{DM} \simeq 2 \times 10^5 M_{\odot}$ and radius $R_{DM} = 10^{-3}$ pc.

present, although they may become important in the future.

As it will be discussed below, the Newtonian effect due to the existence of a sufficiently extended DM sphere around the black hole may cause a apoastron shift in the opposite direction with respect to the relativistic advance due to the black hole. Therefore, we have considered the two effects comparing only the leading terms.

For the DM distribution at the Galactic Center we follow Eq. (3) as done by [1]. Clearly, if in the future faint infrared stars (or spots) closer to the black hole with respect to the S2 star will be monitored [30], this simplified model might well not hold and higher order relativistic corrections may become necessary.

For a spherically symmetric mass distribution (such as that described above) and for a gravitational potential given by Eq. (4), Eq. (5) may be rewritten in the form (see for details [32])

$$\frac{d\mathbf{v}}{dt} \simeq -\frac{GM(r)}{r^3} \left[\left(1 + \frac{4\Phi_N}{c^2} + \frac{v^2}{c^2} \right) \mathbf{r} - \frac{4\mathbf{v}(\mathbf{v} \cdot \mathbf{r})}{c^2} \right], \quad (6)$$

\mathbf{r} and \mathbf{v} being the vector radius of the test particle with respect to the center of the stellar

cluster and the velocity vector, respectively. Once the initial conditions for the star distance and velocity are given, the rosetta shaped orbit followed by a test particle can be found by numerically solving the set of ordinary differential equations in eq. (6).

In Fig. 3, as an example, assuming that the test particle orbiting the Galactic Center region is the S2 star, we show the Post Newtonian orbits obtained by the black hole only, the black hole plus the stellar cluster and the contribution of two different DM mass density profiles. In each case the S2 orbit apoastron shift is given. As one can see, for selected parameters for DM and stellar cluster masses and radii the effect of the stellar cluster is almost negligible while the effect of the DM distribution is crucial since it enormously overcome the shift due to the relativistic precession. Moreover, as expected, its contribution is opposite in sign with respect to that of the black hole [33].

We note that the expected apoastron (or, equivalently, periastron) shifts (mas/revolution), $\Delta\Phi$ (as seen from the center) and the corresponding values $\Delta\phi_E^\pm$ as seen from Earth (at the distance $R_0 \simeq 8$ kpc from the GC) are related by

$$\Delta\phi_E^\pm = \frac{d(1 \pm e)}{R_0} \Delta\Phi, \quad (7)$$

where with the sign \pm are indicated the shift angles of the apoastron (+) and periastron (-), respectively. The S2 star semi-major axis and eccentricity are $d = 919$ AU and $e = 0.87$ [3].

In Fig. 4, the S2 apoastron shift as a function of the DM distribution size R_{DM} is given for $\alpha = 0$ and $M_{DM} \simeq 2 \times 10^5 M_\odot$. Taking into account that the present day precision for the apoastron shift measurements is of about 10 mas, one can say that the S2 apoastron shift cannot be larger than 10 mas. Therefore, any DM configuration that gives a total S2 apoastron shift larger than 10 mas (in the opposite direction due to the DM sphere) is excluded. The same analysis is shown in Figs. 5 and 6 for two different values of the DM mass distribution slope, i.e. $\alpha = 1$ and $\alpha = 2$, respectively. In any case, we have calculated the apoastron shift for the S2 star orbit assuming a total DM mass $M_{DM} \simeq 2 \times 10^5 M_\odot$. As one can see by inspecting Figs. 4-6, the upper limit of about 10 mas on the S2 apoastron shift may allow to conclude that DM radii in the range about $10^{-3} - 10^{-2}$ pc are excluded by present observations.

We notice that the results of the present analysis allows to further constrain the results of the Hall and Gondolo paper [1] who have concluded that if the DM sphere radius is in the range $10^{-3} - 1$ pc, configurations with DM mass up to $M_{DM} = 2 \times 10^5 M_\odot$ are acceptable.

The present analysis shows that DM configurations of the same mass are acceptable only for R_{DM} out the range between $10^{-3} - 10^{-2}$ pc, almost irrespectively of the α value.

IV. CONCLUSIONS

In this paper we have considered the constraints that the upper limit (presently of about 10 mas) of the S2 apoastron shift may put on the DM configurations at the galactic center considered by Hall and Gondolo [1].

When (in about 10-15 years, even without considering improvements in observational facilities) the precision of S2 apoastron shift will be about 1 mas (that is equal to the present accuracy in the S2 orbit reconstruction) our analysis will allow to further constrain the DM distribution parameters. In particular, the asymmetric shape of the curves in Figs. 4-6 imply that any improvement in the apoastron shift measurements will allow to extend the forbidden region especially for the upper limit for R_{DM} .

In this context, future facilities for astrometric measurements at a level 10 μ as of faint infrared stars will be extremely useful [30] and they give a chance to put even more severe constraints on DM distribution.

In addition, it is also expected to detect faint infrared stars or even hot spots [34] orbiting the Galactic Center. In this case, consideration of higher order relativistic corrections for an adequate analysis of the stellar orbital motion have to be taken into account.

In our considerations we adopted simple analytical expression and reliable values for R_{DM} and M_{DM} parameters following [1] just to illustrate the relevance of the apoastron shift phenomenon in constraining the DM mass distribution at the Galactic Center. If other models for the DM distributions are considered (see, for instance [35] and references therein) the qualitative aspects of the problem are preserved although, of course, quantitative results on apoastron shifts may be different.

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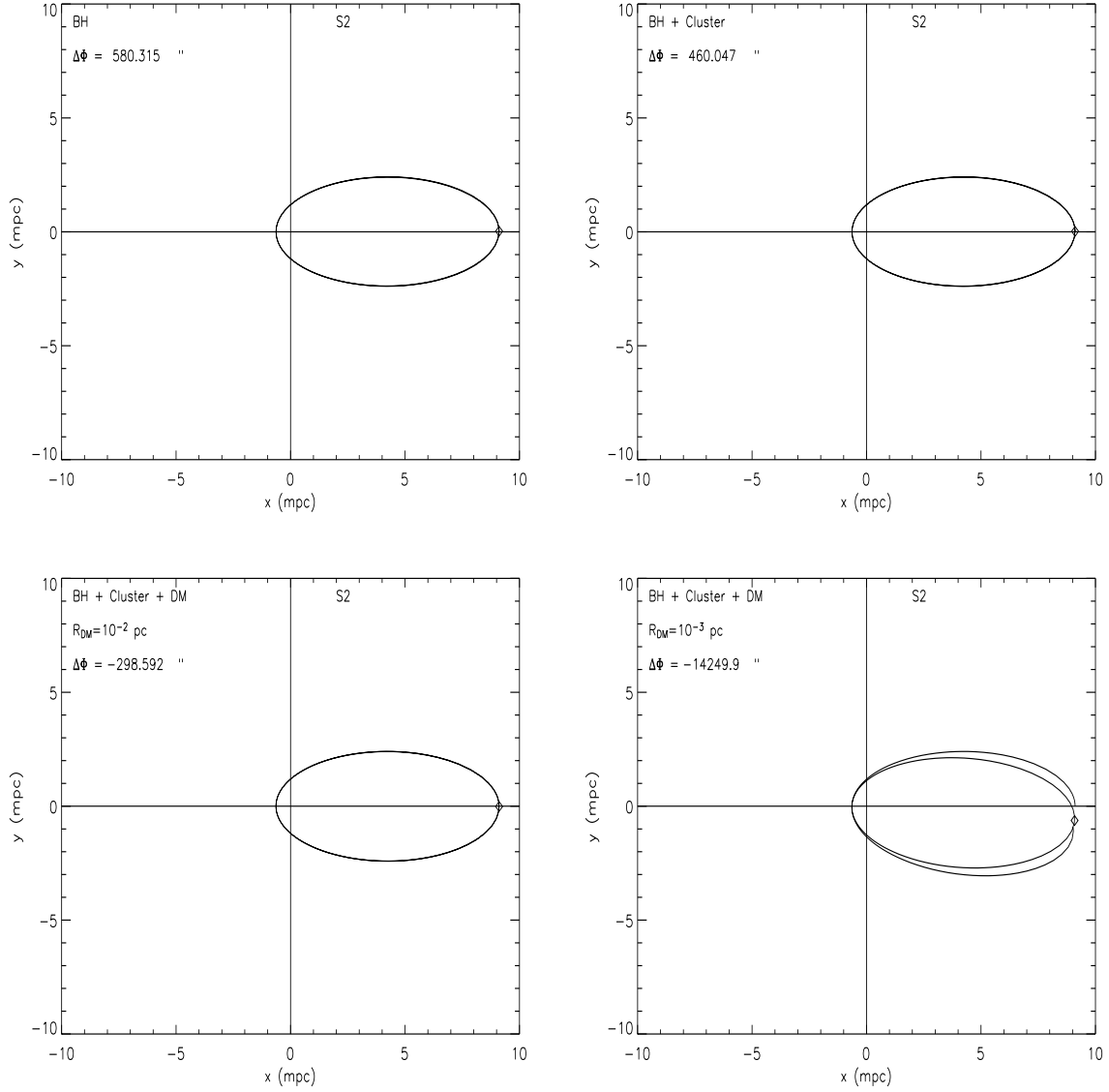


FIG. 3: PN-orbits for different mass configurations at the Galactic Center. The S2 star has been considered as a test particle and its apoastron shift is indicated in each panel as $\Delta\Phi$ (in arcsec). The top-left panel shows the central black hole contribution to the S2 shift that amounts to about 580 arcsec. The top-right panels shows the combined contribution of the black hole and the stellar cluster (taken following eq. 2) to the S2 apoastron shift. In the two bottom panels the contribution due to two different DM mass-density profiles is added (as derived in eq. 3). We assume that DM mass $M_{DM} \simeq 2 \times 10^5 M_{\odot}$.

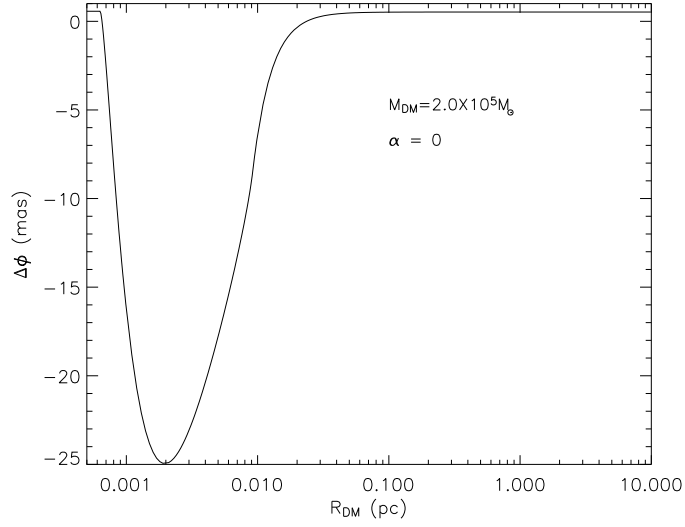


FIG. 4: Apoastron shift as a function of the DM radius R_{DM} for $\alpha = 0$ and $M_{DM} \simeq 2 \times 10^5 M_{\odot}$. Taking into account present day precision for the apoastron shift measurements (about 10 mas) one can say that DM radii R_{DM} in the range $8 \times 10^{-4} - 10^{-2}$ pc are not acceptable.

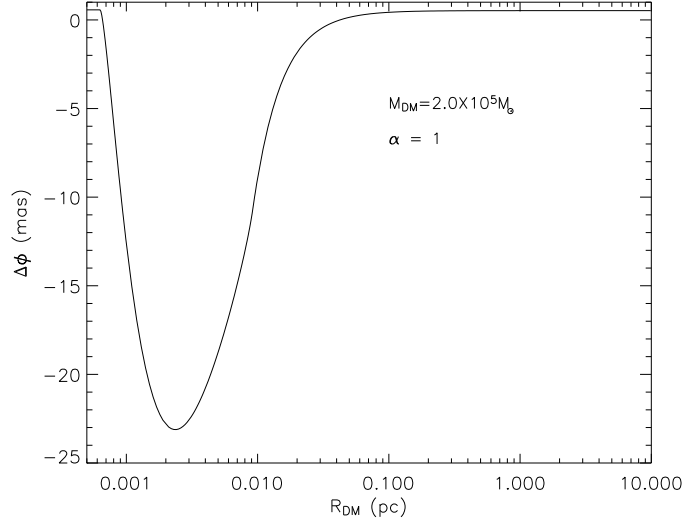


FIG. 5: The same as in Fig. 4 for $\alpha = 1$ and $M_{DM} \simeq 2 \times 10^5 M_{\odot}$. As in the previous case one can say that the S2 apoastron shift put severe limits on the DM mass radii that are not acceptable in the range $9 \times 10^{-4} - 10^{-2}$ pc.

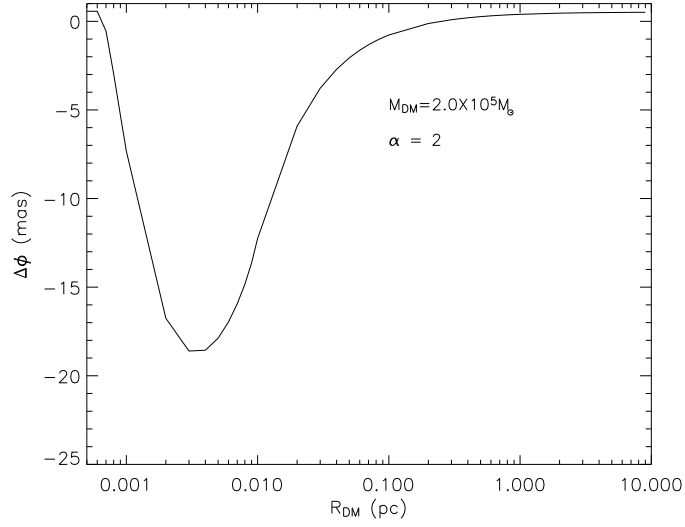


FIG. 6: The same as in Fig. 4 for $\alpha = 2$ and $M_{DM} \simeq 2 \times 10^5 M_{\odot}$. As in the previous case one can say that the upper limit to the S2 apoastron shift allows to constrain the DM radius to be out the range $1.0 \times 10^{-3} - 1.1 \times 10^{-2}$ pc.

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- [38] We want to note that the periastron and apoastron shifts $\Delta\Phi$ as seen from the orbit center have the same value whereas they have different values as seen from Earth (see Eq. (7)). When we are comparing our results with orbit reconstruction from observations we refer to the apoastron shift as seen from Earth.
- [39] Obviously, to take into account gravitational radiation reaction, at least a 2.5 PN approximation is needed (see e.g. [37]).